Abstract
This paper breaks down the causes of variation in mechanical parts into its constituents. It then distinguishes the variation in other processes from plastic parts where the material selection, design rules, tooling and processing have a much greater effect on variation. Once these differences are understood, two very powerful CAD based tools are introduced to be used to minimize the tolerance build up.

Introduction
Let us start with the basics. Tolerance build up is caused by variation which is a part of nature. As examples, this variation can be:

The variation can be in the time it takes to complete a task such as
- Driving to work from home
- Errors in completing a task such as
  - Bags misplaced by airlines
- Dimensions and features within a part in a production process

Using the example of driving to work,
- Normal variation
  - Change in the wind speed and direction
  - Slight variation in the exact path to work (driving on the far or near side of a bend in the road)
  - Effect of other vehicles on the road (being stuck behind a slow-moving vehicle)
  - Number of green and red lights at intersections
  - Error in reading the elapsed time
  - Rounding off minutes in the departure and arrival time
- Special causes
  - Accident on the road
  - Snow or ice
  - Car trouble, driving slower because of low pressure in one of the tires

Ideal vs. Real World
In an ideal world, all the dimensions in all the parts within an assembly would be exactly nominal and the assemblies would fit perfectly all the time as in the top figure. In real life though the distribution looks more like the bottom figure.

Root Causes of Variation in Mechanical Processes
The cause for variation in mechanical process can be divided in to:

- Constant Factors
- Random Variation
- Setup Variation
- Special Causes
- Measurement Error

Constant Factors
• Initial Error
  o Multiple tools and cavities
    ▪ Each cavity or tool will have slightly different dimensions
  o Tooling
    ▪ Wrong initial dimensions – distance between two pins in a stamping die is 1.05 inches instead of 1.00 inches. The distance between the centers of the holes will be .05 inch longer – always!
  o Wrong shrinkage factor – plastic part
    ▪ Wrong shrinkage used - .008 inch/inch instead of .006 inch/inch. A 10-inch-long part will be .002 * 10 = .02 inch longer – always!
  o Wrong spring back calculation – see figure 1
    ▪ Wrong spring back compensation in the die – dimension A will be A + Y * tan 5 deg -always!

Figure 1

Tool Dependent Errors – Regardless of Age
• Dimensions across parting line. See Figure 2. All dimensions marked will vary based on the alignment of the two halves of a plastic or diecasting mold.

Figure 2

• Dimensions created by moving parts within one half of the tool. See Figure 3. The actual height of the snap feature depends on the exact position of the lifter from shot to shot.

Figure 3 (1)

• Depth of stroke (2). See Figure 4. The angle of the bend and other dependent dimensions will vary based on the down stroke of the punch every hit.

Figure 4 (2)

• Progression in a stamping die (3). See Figure 5. The strip may be underfed or overfed on successive hits – resulting in variations in the features within a stamping.
In Figure 6, the exact location of the hole will depend on the position of the part within the fixture. This may change due to slop in the positioning of the work piece or a burr on it.

**Tool Dependent Tolerance – As Tool Ages**

- Dimensions across parting lines
  - As an example, an insert to repair a damaged parting line may be slightly higher than the rest of the parting line causing a change in all dimensions across the parting line
- Dimensions created by moving parts within one half of the tool
  - As an example, the height of the snap may grow as the lifter wears
- Depth of stroke
  - As an example, a punch may wear causing the bend angle in the stamping to change
- Progression in a stamping die
- A worn or damaged pilot punch may cause the progression of the stamping to vary - impacting multiple dimensions
- Tool Wear
  - Machined parts will continue to change as the cutting tools wear and are replaced

**Set Up**

- Almost all processes will have some variation in dimensions from set up to set up.
  - In the case of molded parts there may be variation in the pressure, temperature and time settings (more details later)
  - In the case of stampings, the down stroke depth may vary
  - In the case of machining, the part placement, cutting tools and depth of cutting tools may vary

**Material Related - Plastics**

- Shrinkage
  - The shrinkage varies slightly from lot to lot
- Directional Shrinkage
  - The directional shrinkage varies both from lot to lot and within a part – as an example the density and orientation of glass concentration can vary from part to part and lot to lot. The part will shrink less in the direction of flow where the fibers are also similarly aligned. If the fibers are aligned in a direction at right angle to the flow, the shrinkage will be more. Exact concentration and alignment of the fibers will vary from part to part. See Figure 7.
Material Related – Metals

- Temper, Spring Back
  - The temper and therefore the spring back may vary slightly from lot to lot
- Temper – Distortion
  - Parts that are heat treated or annealed after forming or machining may show uneven distortion such as in Beryllium Copper alloys during the post stamping heat treatment
- Material Thickness
  - The thickness variation in the metal strip may cause variations in spring back causing shifts both in angles and feature dimensions

Plastics - Material, Design, Tooling and Process Interdependence on Variation

Plastics are very different from metals
- Much more interdependence of materials, design, tooling and processing
- Design errors may cause variations during processing that metals may have relatively minor effect from
- Examples
  - Design
    - Wall thickness uniformity
      - Recommended - amorphous materials no more than 25% variation
      - Semi Crystalline materials have no more than 15% variation
    - Thin to thick flow
    - Material
      - Shrinkage
      - MFI (Melt Flow Index)
    - Tooling
      - Gate and runner design and placement

Material Example – Plastics

Figure 8 shows a nylon (left) and a Polycarbonate part (right) molded from the same mold. The shrinkage in the case of nylon is less in the direction of flow compared to its cross direction. Therefore, the part shrinks less in the radial direction than the circumferential direction resulting in the oil canning effect. The PC part with its uniform shrinkage in every direction does not show this phenomenon.

Figure 8

Design Example – Plastics – Non-Uniform Wall

See Figure 9 and 10. The left of the part is half as thick as the right. Plastic being a good heat insulator, the temperature of the left side is 118 deg C compared to 238 deg C on the right when the part is ejected. The right continues to shrink for a long time after the left has cooled down. This results in warpage at the interface.
Design Example – Plastics – Hesitation

See Figure 11. A thick section surrounds a thin section. The thick surrounding area fills first as the path of least resistance. By the time the thick section is filled, the plastic has lost both its temperature and pressure causing incomplete filling of the thin section. This uneven packing of the adjacent sections will lead to part to part variation in shrinkage and therefore dimensions.

Figure 11

Plastics – Effect of Gate and Runners

See Figures 12 and 13 with two gate and runner arrangements for the same part. In Figure 12 there is a big loss of pressure in the runner system and non-uniform filling of the part. Figure 13 shows a much less pressure drop and much more uniform filling. Because of this the part will shrink more non-uniformly for the first runner system than the second one. The second part will have a lot more part to part uniformity in dimensions because of this.

Figure 12
Process Related Tolerance – Plastics

The following shows most of the variations in parameters in processing a plastic part that will cause non-uniform packing, filling cooling and crystallinity in the case of semi-crystalline parts resulting in post molding distortion and change in dimensions.

- **Pressure, Temperature, Time, Humidity**
  - Pressure
    - Injection Pressure
    - Holding Pressure
    - Back Pressure
  - Temperature
    - Melt temperature
    - Coolant temperature and flow rate
    - Mold temperature
    - Drying temperature
  - Time
    - Injection Time
    - Gate Freeze Time
    - Hold Time
    - Decompression
    - Drying Time
  - Humidity
    - Dew point of drying air

Measurement Error
- **Reproducibility**
  - Two operators or two setups coming up with different readings

Overall Tolerance Equation
Based on the forging discussion the overall tolerance equation can be defined as:

\[ T_{Total} = T_{Material} + T_{Design} + T_{Tooling} + T_{Setup} + T_{Processing} + T_{Measurement} \]

Process Capability
Process capability can be defined as the ability of a process to produce output within specification limits for processes that are in a state of statistical control. The terms used to define the process capability are

\[ C_p = \frac{\text{Spec width}}{\text{Natural tolerance}} = \frac{\text{Upper spec} - \text{Lower spec}}{6\sigma} \]

\[ C_{pk} = \text{Lesser of } \frac{(\text{USL} - \text{mean})}{3 \sigma} \text{ or } \frac{(\text{mean} - \text{LSL})}{3 \sigma} \]

Figure 14 illustrates processes with various degree of process control. The left target has both precision and low variation. The middle has good accuracy but less precision. The third one has good precision but not good accuracy.

Six Sigma Tolerancing
In the eighties and early nineties, Motorola engineers were not satisfied with the then accepted 3 sigma variation which would result .27% dimensions being out of specification. Therefore, they decided to
reduce the process variation to HALF the design tolerance (i.e. \( \text{Cp} = 2.0 \)) while allowing the mean to shift as much as 1.5\( \sigma \) from the target. The resulting area under the shifted curves beyond the six sigma ranges (i.e. the tolerance limits) is only 0.0000034, or **3.4 ppm**. See figure 15.

![Six-Sigma Quality](image)

**Figure 15.**

**Causes and Cost of Poor Process Capability**

Process capability will be significantly reduced by

- Poor or sloppy tolerance stack up analysis
- Poor material selection
- Poor design
- Less than optimized tooling and processing

This in turn will significantly increase the cost of the product and reduce the revenue by

- Unnecessarily high cost of tooling
- Resorting to selective assembly
- High cost of scrap and reassembly
- Addition of non-value added operations like inspection, testing, re-work and retesting
- Warranty costs
- Recalls
- Opportunity costs due to poor perception of quality by the customer

**What the Designer Can Do to Increase the Process Capability**

- Avoid design errors that will reduce the process capability
- Understand the process capability and use realistic tolerances
- Adjust and optimize tolerance stack up based on the process capabilities of all the components in the chain
  - As an example, in a chain consisting of a large plastic part and a sheet metal part, it may be more practical to tighten the tolerances on the holes on the metal stamping as opposed to posts on the plastic part that fit inside those holes
- Systematically and uniformly calculate the total tolerance stack up

**Tolerance Optimization and Analysis**

Design engineers, while considering dimensional variations, need to look at both part level tolerances and the overall impact of individual part tolerance on the overall assembly. Hence tolerance stack-up analysis popularly known as TSA is an iterative process. Design engineer will first assign tolerance on the individual part looking at various factors such as manufacturability and cost along with basic form, fit and function of the part. After performing TSA, individual part tolerances may need to be relaxed or tightened depending on analysis results of the stack-up analysis.

Designer will typically assign tolerances to major dimensions in the part which are important from functional point of view and then manufacturing will have to ensure that those requirements are adhered to during production process. A typical way to represent these tolerances are shown in the figure below (figure 16)

![Figure 16 - Typical way to represent key tolerances on individual part](image)

**Design for Manufacturing and Dimensional Variations in Part**

While looking at dimensional variations, it is utmost important to ensure that part is manufacturable. More often than not, TSA performed on parts where manufacturability analysis is not considered will fail miserably not only in manufacturing and assembly but also from the performance point of view. DFM practices are often linked to the way a ‘feature’ is designed and it provides a mechanism to control them, it does help in avoiding variation in part
dimensions and also a mechanism to compensate for such variation in terms of designing such features. Some of the common DFM guidelines for plastics process are – rib height to wall thickness ratio, core hole radius in boss, wall thickness variation, bottom radius for lip feature, etc.

DFMPro is a design assistant from Geometric which will provide feedback to designer right within CAD environment and will provide DFM feedback on features in the design that needs to be controlled to reduce variation in design (figure 17).

Tolerance Stack Up Analysis
Post DFM analysis, every part will have its critical dimension assigned with tolerance and then the same needs to be studied for verifying the overall impact on variation in the assembly. There are two most commonly followed methods for this analysis –
  a) Worst case analysis
  b) Statistical analysis
The purpose of the stack-up analysis is to control the input tolerances (min and max) to achieve better yield and to make sure that the variation fits between the desired upper and lower spec limits of assembly. Mathematical tolerance stack-ups use the worst-case maximum of dimensions and tolerances to calculate the maximum and minimum distance between two features or parts. Statistical tolerance stack-ups evaluate the maximum and minimum values based on mathematical calculation combined with some method for establishing likelihood of obtaining the maximum and minimum values, such as Root Sum Square (RSS) or Monte-Carlo methods.

To achieve the desired results, a designer must look at the major contributors of tolerance. This goes by a simple 80-20 rule that more often there are less number of parts which would cause greater variation in design so that minimum part tolerances should be modified reducing overall design effort. In example below (Fig. 19), by adjusting tolerances of DIM4, it may be possible to achieve desired outcome – yield for the process.

By tightening the tolerance of DIM4 from ± 0.2 to ± 0.1 the overall yield goes up significantly and the modified distribution chart looks more promising. (Figure 20)
While adjusting the tolerances for dimensions, it is important to understand association of those dimensions with the part type. For example, it would be very difficult to have a tolerance of ±0.001 to be achieved on a large plastic injection molded part vis-à-vis a machined steel part. Also, tightening of dimensions may impact manufacturability and increase cost of manufacturing.

Geometric Dimensioning and Tolerancing further provides information to downstream manufacturing departments about the degree and accuracy of the precision that needed to be controlled for feature of the part. ASME and ISO provides typical standards use in industry for GD&T. Impact of GD&T and assembly shift for mechanisms like fasteners, holes and pins needs to be further considered while performing TSA.

References and Acknowledgements

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6. Courtesy http://www.slideshare.net/IVT_Network/introduction-to-statistical-applications-for-process-validation
7. Courtesy http://scialert.net