NUMERICAL SIMULATION OF DISCONTINUOUS SLOW CRACK GROWTH OF SEMI-ELLIPtical SURFACE CRACK IN POLYETHYLENE BASED ON CRACK LAYER THEORY

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Abstract

For the structural application of engineering thermoplastics, the knowledge of failure modes depending on their service conditions is essential. The most prevalent failure mode is brittle fracture followed by the slow crack growth (SCG) initiated by surface flaws. In that the general geometry of the surface flaws is semi-elliptical, it is vital to investigate the SCG aspects from such kind of shape. The simple strategy which has been employed to predict the crack growth aspect is an application of conventional law, Paris-Erdogan relationship. The approach is regarded as quite simple since only stress intensity factor (SIF) is needed for a crack driving force term. However, through this empirical relationship, the SCG in engineering thermoplastics cannot be properly modeled. For example, in case of the high-density polyethylene (HDPE), frequently used for water transportation pipelines, the crack usually propagates discontinuously. It arises from the existence of a significant damaged zone in front of the main crack tip, which is normally observed in engineering thermoplastics. Thus, adopting one linear elastic fracture mechanics (LEFM) parameter may not reflect the severe damage zone. To handle this feature properly, a theoretical approach with a reflection of such energy dissipation is necessary. In this study, the crack layer (CL) theory was employed to simulate the discontinuous SCG of semi-elliptical surface crack in HDPE plate with finite thickness. The existing 1-dimensional CL theory was expanded to the semi-elliptical crack growth.

Introduction

The application area of engineering thermoplastics has been more broaden, due to the modified polymerization and manufacturing processes. Given that the application area of engineering thermoplastics has been expanded to the load-bearing components, the researches on the lifetime prediction of those materials are necessary.

For instance, high-density polyethylene (HDPE) is usually applied to the water transportation pipelines. There are mainly three failure modes in HDPE pipes, depending on the applied load level. In condition of the severe hoop stress, the hydro-bursting occurs with localized buckling. In the middle load level, the brittle fracture caused by slow crack growth (SCG) becomes a main failure mode [1]. The stress corrosion cracking (SCC) can be also observed along the excessive contact period with aggressive fluids [2]. The SCC usually occurs in low load level. The representative field failure mechanism of HDPE pipes is the brittle fracture in an intermediate load level. It is well-accepted that the SCG causing brittle fracture is quasi-static process [3].

HDPE shows different SCG modes, regarding the test conditions, load level and temperature. The continuous, discontinuous, and combined mode of SCG can be observed. Thus, the different SCG mechanisms can be marked down according to both conditions. Such SCG mechanism map is called DBT2 concept [1]. The distinguished SCG modes cannot be simulated by applying the Paris-Erdogan relationship, employing the conventional linear-elastic fracture mechanics parameter. Because the thermoplastics usually reveal a significant damaged region, so called the process zone (PZ) in front of the crack tip, the interaction between the PZ and main crack must be considered. In the crack layer (CL) theory, the PZ and main crack are considered as a system, CL. Through the CL theory, the interpretation of thermodynamic interaction between PZ and main crack enables us to simulate the discontinuous SCG [4, 5]. However, the researches on the CL growth simulation have been focused on the 1-dimensional SCG. Considering that the initially generated surface cracks are normally semi-circular or semi-elliptical shape, the 2-dimensional CL model applicable to these surface crack is needed.

In this study, the current CL theory was modified to semi-elliptical surface crack in HDPE plate. The thermodynamic forces (TF) for crack and PZ growth of semi-elliptical crack were obtained. By applying the linear irreversible thermodynamics, the discontinuous SCG for the surface crack was simulated.

Backgrounds
The crack layer (CL) is a system of main crack and surrounding damage zone, so called process zone (PZ). The feature of PZ depends on the kinds of thermoplastics. The high-density polyethylene (HDPE) reveals the narrow wedge shape PZ, composed of highly drawn fibrils, with a clear boundary. Many researches regarding the CL growth modeling of HDPE have been conducted due to these favorable features [3, 5, 6]. Let us consider the HDPE plate with a semi-elliptical surface crack under constant tensile remote stress $\sigma_w$ (Figure 1). The CL of the semi-elliptical surface crack is illustrated in Figure 2. We assume the crack and PZ front as semi-elliptical profile. Therefore, providing that the crack or PZ size along the deepest (A) and surface (C) direction, $x$ and $y$ coordinate respectively in Figure 1, the shape of crack and PZ can be determined.

The TF for crack growth on the deepest position A and surface one C are given by

$$X_{A}^{CR} = J_{1,A}^{CR} - 2\gamma_{A}$$
$$X_{C}^{CR} = J_{1,C}^{CR} - 2\gamma_{C}$$

where the $J_{1,A}^{CR}$ and $J_{1,C}^{CR}$ are the energy release rate (ERR) at the crack tip, on the deepest point (A) and surface point (C), respectively. The $2\gamma$ is the surface fracture energy (SFE) at each position. Similarly, the TF for PZ growth on both position can be expressed by following equations.

$$X_{A}^{PZ} = J_{1,A}^{PZ} - \gamma^{s} \frac{\partial V_{PZ,A}}{\partial A_{A}^{PZ}}$$
$$X_{C}^{PZ} = J_{1,C}^{PZ} - \gamma^{s} \frac{\partial V_{PZ,C}}{\partial A_{C}^{PZ}}$$

where the $J_{1,A}^{PZ}$ and $J_{1,C}^{PZ}$ are the energy release rate (ERR) at the PZ tip, on the deepest point (A) and surface point (C), respectively. The $\gamma^{s}$ is the required cavitation energy per unit volume, to transform of undamaged material into the PZ matter (Figure 3). The $V_{PZ}$ is volume of PZ on considered segments. $A_{PZ}$ is PZ area illustrated in Figure 2. Finally, the kinetics of crack and PZ growth can be calculated by linear irreversible thermodynamics [4, 7].

$$\frac{dl_{A}}{dt} = \kappa_{A}^{CR} X_{A}^{CR}, \quad \frac{dl_{C}}{dt} = \kappa_{C}^{CR} X_{C}^{CR},$$
$$\frac{dL_{A}}{dt} = \kappa_{A}^{PZ} X_{A}^{PZ}, \quad \frac{dL_{C}}{dt} = \kappa_{C}^{PZ} X_{C}^{PZ}$$

where the $\kappa_{A}^{CR}$ and $\kappa_{A}^{PZ}$ are kinetic coefficients for crack and PZ growth, respectively.
Results and discussions

The input parameters in the CL growth simulation of semi-elliptical surface crack in high-density polyethylene (HDPE) plate are provided in Table 1. $E'$ is plane strain elastic modulus, $W$ the width of plate, and $B$ the thickness of plate. The transformation energy density $\gamma^{tr}$ in equation 2 is inserted as 30 mJ/mm$^3$, in accordance with the reported value for HDPE in room temperature [1]. The initial crack depth $c$ is constant as 2 mm. The three crack lengths $2a$ of 4, 10, 16 mm are considered to validate the model. As shown in Figure 4-6, the proposed CL model can simulate the discontinuous slow crack growth (SCG) pattern, which is frequently observed in HDPE [8, 9].

Just after the application of tensile load, the PZ would be developed to achieve the thermodynamic equilibrium (equation 2). The saturation of PZ length indicates the zero TF. Also, the PZ material should be degraded with the elapsed time, due to the creep. Such degradation is manifested by the decrease in surface fracture energy (SFE), $2\gamma$. Once the TFs for crack growth become positive, the crack starts to grow according to the equation 1 and 3. In case of initial aspect ratio of 0.25, the crack at the deepest position (Figure 4a) jumps earlier than at the surface (Figure 4b). After the crack jumps at the deepest position, the PZ size is increased on the surface position to achieve thermodynamic equilibrium, indicating the zero TF. That is, the CL system of semi-elliptical surface crack can be considered as 4 degree of freedom (DOF), with the influence between the components. Figure 5 and 6 similarly reveal the discontinuous CL growth patterns for different initial crack aspect ratios.

Figure 7 shows the variation of crack aspect ratio, with the elapsed time. At the shallow crack case ($\alpha_0 = 0.25$), the crack at deepest position may propagate faster than surface position in initial period. Therefore, the aspect ratio of crack increases consistently. On the contrary, for the semi-circular crack case ($\alpha_0 = 1.0$), the aspect ratio is decreased slightly. It should be noted that in three cases, the aspect ratios of crack converge into about 0.7, indicating the stable shape. Such convergent tr-

Table 1. Input parameters for CL growth simulation of semi-elliptical surface crack.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$2a$</td>
<td>mm</td>
<td>4, 10, 16</td>
</tr>
<tr>
<td>$c$</td>
<td>mm</td>
<td>2</td>
</tr>
<tr>
<td>$W$</td>
<td>mm</td>
<td>100</td>
</tr>
<tr>
<td>$B$</td>
<td>mm</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_\infty$</td>
<td>MPa</td>
<td>8</td>
</tr>
<tr>
<td>$E'$</td>
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</tr>
<tr>
<td>$\gamma^{tr}$</td>
<td>mJ/mm$^3$</td>
<td>30</td>
</tr>
</tbody>
</table>
Figure 7. The variation of crack aspect ratio with elapsed time.

ends regardless of initial crack aspect ratio have been reported in numerous literatures [10-12]. Therefore, the proper tendencies which have been proven through several methods can be simulated by the suggested CL model.

Conclusions

In order to estimate the unique slow crack growth (SCG) mode in HDPE, the fundamental approaches taking the significant damage zone into account are essential. The discontinuous SCG from semi-elliptical surface crack in HDPE plate under uniform tensile stress was simulated by developing 2-dimensional crack layer (CL) theory. The existing CL theory was expanded to semi-elliptical crack. The discontinuous SCG of the surface crack successfully simulated by the developed model.

References